## Chapter 12

## Surface Area and Volume

Section 1
Exploring Solids

## GOAL 1: Using Properties of Polyhedra

A polyhedron is a solid that is bounded by polygons, called faces, that enclose a single region of space. An edge of a polyhedron is a line segment formed by the intersection of two faces. A vertex of a polyhedron is a point where three or more edges meet. The plural of polyhedron is polyhedra, or polyhedrons.


## Example 1: Identifying Polyhedra

Decide whether the solid is a polyhedron. If so, count the number of faces, vertices, and edges of the polyhedron.
a.


Yes
Faces: 5
Vertices: 6
Edges: 9
b.


No


Yes
Faces: 7
Vertices: 7
Edges: 12

Of the five solids below, the prism and pyramid are polyhedra.
The cone, cylinder, and sphere are not polyhedra.


Prism


Cylinder


Pyramid


Cone


Sphere

A polyhedron is regular if all of its faces are congruent regular polygons. A polyhedron is convex if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron. If this segment goes outside the polyhedron, then the polyhedron is nonconvex, or concave.

regular, convex

nonregular, nonconvex

## Example 2: Classifying Polyhedra

Is the octahedron convex? Is it regular?
a.


Convex, regular
b.


Convex, not regular
c.


Concave, not regular

Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a cross section. For instance, the diagram shows that the intersection of a plane and a sphere is a circle.


## Example 3: Describing Cross Sections

Describe the shape formed by the intersection of the plane and the cube.
a.


pentagon
c.

triangle

## GOAL 2: Using Euler's Theorem

There are five regular polyhedra, called Platonic solids, after the Greek mathematician and philosopher Plato. The Platonic solids are a regular tetrahedron (4 faces), a cube ( 6 faces), a regular octahedron (8 faces), a regular dodecahedron ( 12 faces), and a regular icosahedron ( 20 faces).


Regular tetrahedron 4 faces, $\mathbf{4}$ vertices, $\mathbf{6}$ edges


Cube
6 faces, 8 vertices, 12 edges


Regular octahedron 8 faces, 6 vertices, 12 edges


Regular dodecahedron 12 faces, 20 vertices, 30 edges


Regular icosahedron 20 faces, 12 vertices, 30 edges

Notice that the sum of the number of faces and vertices is two more than the number of edges in the solids above. This result was proved by the Swiss mathematician Leonhard Euler (1707-1783).

## THEOREM

theorem 12.1 Euler's Theorem
The number of faces $(F)$, vertices $(V)$, and edges $(E)$ of a polyhedron are related by the formula $F+V=E+2$.

Example 4: Using Euler's Theorem edges The solid has 14 faces; 8 triangles and 6 octagons. How many vertices does the
solid have?

$$
\begin{gathered}
8(3)+W(8)=72 \div 2=36 \\
F+V=E+2 \\
14+V=36+2 \\
M+V=38 \\
V=24
\end{gathered}
$$



## Example 5: Finding the Number of Edges

Chemistry In molecules of sodium chloride, commonly known as table salt, chloride atoms are 8 faces arranged like the vertices of regular octahedrons. In the crystal structure, the molecules share edges. How many sodium chloride molecules share the edges of one sodium chloride molecule?


$$
\begin{gathered}
F+V=E+2 \\
F+6=E+2 \\
14=E+2 \\
12=E
\end{gathered}
$$

Example 6: Finding the Number of Vertices
Sports A soccer ball resembles a polyhedron with 32 faces; 20 are regular hexagons and 12 are regular pentagons. How many vertices does this polyhedron have?

$$
\begin{aligned}
& 20(6)+12(5)=180 \div 2=90 \\
& F+V=E+2 \\
& 32+V=90+2 \\
& 32+V=92 \\
& V=60
\end{aligned}
$$

EXIT SLIP

